The ρ-Meson as a Collective Excitation*

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Abstract

A model of the ρ-meson as a collective excitation of $q\bar{q}$ pairs in a system that obeys the modified Nambu–Jona-Lasinio Lagrangian is proposed. The ρ emerges as a dormant Goldstone boson. The origin of the ρ-meson mass is understood as a result of spontaneous chiral symmetry breaking. The low-energy dynamics of ρ, π, ω and γ is consistently described in this new framework. The model accounts for the origin of the celebrated Kawarabayashi–Suzuki–Riazuddin–Fayyazuddin relation.

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If we assume that pions are bound states of the quark and antiquark, one can explain the small pion mass by analogy with superconductivity, considering these ground-state mesons to be collective excitations of $q\bar{q}$ pairs. Nambu and Jona-Lasinio (NJL) have done it in their pioneering work [1]. In the NJL model pions appear as massless Goldstone modes associated with the dynamical spontaneous breakdown of chiral symmetry of the initial Lagrangian. An attractive idea is to treat low-lying hadronic states similarly to the pions as collective excitations of the QCD vacuum, their masses resulting from dynamical breaking of chiral symmetry. Is it possible to modify the NJL Lagrangian in such a way as to simulate the above situation at least for the lowest meson states, first of all for the ρ-meson? An encouraging fact is the existence of a relationship between the ρ-meson and pion parameters expressed by the well-known Kawarabayashi–Suzuki–Riazuddin–Fayyazuddin (KSRF) relation [2].

Another principle underlies the frequently used current models describing vector particles on the basis of the extended NJL Lagrangian [3]. The ρ-meson mass is not determined by the quark condensate but fixed *ad hoc* by choosing the four-quark interaction constant. This breaks the integrity of the approach where mass spectra of collective $q\bar{q}$ pair excitations depend only on the order parameter.

A new approach to consideration of vector excitations in the extended NJL model is proposed in the present Letter. It solves the problem of the π–ρ system spectrum. The essential feature of the interpretation of vector mesons which I advocate here is that they are the result of $(\bar{q} \overset{\leftrightarrow}{\partial}_\mu \lambda_a q)^2$ four-quark interactions. The special ansatz relates constants of scalar and vector four-quark vertices. A gap solution appears in the vector channel. This state is described by an antisymmetrical tensor field and has a mass $m_\rho^2 = 10m^2$, where $m \simeq 242$ MeV is the constituent quark mass. It can be identified with the ρ-meson. The dynamics of the state is studied. An interesting consequence of the model is the substantiation of the KSRF relation, which is found here in the limiting case $m_\pi = 0$. Despite the relationship between four-quark interaction constants, the model still allows freedom in choosing the common constant characterizing the coupling between the vector fields and quarks. It can be used to validate the hypothesis of universality of vector meson interactions. The non-Abelian anomaly structure of the effective Lagrangian for pions and spin-one mesons is different from the conventional one,

obtained by Witten's "trial-and-error" gauging of the Wess-Zumino term in the paper by Ö.Kaymakcalan, S.Rajeev and J.Shechter [4]. The model forbids the $\omega\rho\pi$ vertex at the quark one-loop level. The $\omega \to \pi\pi\pi$ decay with the only contact term is well described. Certainly it may be related with the new understanding of the origin of spin-one mesons.

Let us consider the $U(2)$ symmetrical Lagrangian with four-quark interaction of the Nambu–Jona-Lasinio type (with the coupling constant G_1) and additional vector interaction (with the coupling constant H)

$$L(q) = \bar{q}(i\gamma^\mu \partial_\mu - \widehat{m})q + \frac{G_1}{2}\left[(\bar{q}\tau_a q)^2 + (\bar{q}i\gamma_5\tau_a q)^2\right] + \frac{H}{2}(\bar{q}\tau_a \overleftrightarrow{\partial_\mu} q)^2, \qquad (1)$$

where $\bar{q} = (\bar{u}, \bar{d})$ are coloured current quark fields with current mass $\widehat{m}, \tau_a = (\tau_0, \tau_i), \tau_0 = I, \tau_i$ are Pauli matrices of the flavour group $SU(2)_f$. We represent the four-quark interaction constant with a derivative as a product of two quantities of the same dimensionality $H = G_2 G$. The dimensionality of the constants is $[G_\alpha] = M^{-2}$. The derivative $\overleftrightarrow{\partial_\mu}$ is defined as $2\bar{q} \overleftrightarrow{\partial_\mu} q = (\bar{q}\partial_\mu q - \partial_\mu \bar{q} q)$. We do not require here the chiral symmetry for vector interactions. This question will be investigated elsewhere.

With the help of auxiliary meson fields $\bar{\sigma}, \tilde{\pi}, \tilde{\rho_\mu}$, Lagrangian (1) takes the form

$$L' = \bar{q}(i\gamma^\mu \partial_\mu - \widehat{m} + \bar{\sigma} + i\gamma_5 \tilde{\pi} + i\sqrt{G}\tilde{\rho_\mu} \overleftrightarrow{\partial_\mu})q - \frac{\bar{\sigma}_a^2 + \tilde{\pi}_a^2}{2G_1} + \frac{\tilde{\rho}_{\mu a}^2}{2G_2}. \qquad (2)$$

Here $\bar{\sigma} = \bar{\sigma}_a \tau_a, \tilde{\pi} = \tilde{\pi}_a \tau_a, \tilde{\rho}_\mu = \tilde{\rho}_\mu^a \tau_a$, and the derivative $\overleftrightarrow{\partial_\mu}$ affects only quark fields. Owing to the dimensional constant $\sqrt{G}$, the vector field will have the same dimensionality as other meson fields. The vacuum expectation value of the field $\bar{\sigma}_0$ turns out to be different from zero. To obtain the physical field $\tilde{\sigma}_0$ with $< \tilde{\sigma}_0 >= 0$ we perform a field shift resulting in a new quark mass m to be identified with the mass of the constituent quarks: $\bar{\sigma}_0 - \widehat{m} = \tilde{\sigma}_0 - m, \bar{\sigma}_i = \tilde{\sigma}_i$, where m is deduced from the gap equation.

Let us integrate over the quark fields in the generating functional associated with Lagrangian (2). Evaluating the resulting quark determinant by a loop expansion we get

$$L_{coll} = -i\mathrm{Tr}\ln\left[1 + \frac{(\tilde{\sigma} + i\gamma_5\tilde{\pi} + i\sqrt{G}\tilde{\rho}_\mu \overleftrightarrow{\partial_\mu})}{(i\gamma^\mu\partial_\mu - m)}\right]_\Lambda - \frac{\bar{\sigma}_a^2 + \tilde{\pi}_a^2}{2G_1} + \frac{\tilde{\rho}_{\mu a}^2}{2G_2}. \qquad (3)$$

The index Λ indicates that a regularization of the divergent loop integrals is introduced. In this case it is the Pauli–Villars regularization [5]. It conserves vector gauge invariance and at the same time reproduces the quark condensate and the current quark mass. The Pauli–Villars cut-off Λ is introduced by replacements $\exp(-izm^2) \to R(z)$ and $m^2 \exp(-izm^2) \to iR'(z)$, where $R(z)$ is the regularizing function that satisfies the conditions $R(0) = R'(0) = \ldots = 0$.

The effective Lagrangian (3) allows us to calculate the leading low-energy behaviour of any N-point meson function. This behaviour is governed by the first terms in the expansion of derivatives [6]. From the requirement for the terms linear in $\tilde{\sigma}$ to vanish we get a gap equation

$$m - \widehat{m} = 8mG_1I_1 \tag{4}$$

(see Eq.(6) for notation). The effective meson Lagrangian describing scalars and pseudoscalars is well known [7], so we turn to the effective Lagrangian of the vector fields. To the order of p^4 the two-point function $< 0|T\tilde{\rho}_\mu\tilde{\rho}_\nu|0 >= T_{\mu\nu}$ is given by

$$T_{\mu\nu}(p) = g_{\mu\nu}(G_2^{-1} - GI_0) + \frac{GI_2}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})(p^2 - 10m^2) + \mathcal{O}(p^6). \tag{5}$$

Here we use the notation

$$I_\alpha = -\frac{3c_\alpha}{16\pi^2}\int_0^\infty \frac{dz}{z^{3-\alpha}}R(z), \tag{6}$$

where $\alpha = 0,\, 1,\, 2, c_0 = 20, c_1 = i, c_2 = -1$.

Let us introduce the ansatz $1 - HI_0 = 0$. On the one hand, it ensures gradient invariance of self-energy (5) and on the other it relates G_1 and H constants by the condition $HI_0 = 8G_1I_1 + \widehat{m}/m$ via gap equation (4). It was stressed by T.Eguchi [8] that one can create a local gauge symmetry starting from global invariance in nonlinear spinor theories. In Lagrangian (2) we have a collective excitation $\tilde{\rho}^a_\mu$ which is coupled to a conserved current $(\bar{q} \overleftrightarrow{\partial}_\mu \tau_a q)$. Hence (2) is invariant under $\tilde{\rho} \to \tilde{\rho} + \partial\alpha$ with an arbitrary $\alpha(x)$. Although the term $\tilde{\rho}^2_{\mu a}/2G_2$ seems to spoil this invariance, it in fact eliminates the gauge-noninvariant part coming from radiative corrections and preserves the gauge invariance.

Renormalizing the ρ-meson field $\tilde{\rho} = \sqrt{3/I_2}\rho$ we arrive at the Lagrangian

$$L_\rho = G\left(-\frac{1}{2}\partial_\mu \rho^a_{\mu\nu}\partial_\lambda \rho^a_{\lambda\nu} + \frac{1}{4}m_\rho^2 \rho^a_{\mu\nu}\rho^a_{\mu\nu}\right), \tag{7}$$

where the ρ-meson mass is expressed in terms of the constituent quark mass

$$m_\rho^2 = 10m^2. \tag{8}$$

Thus, like the mass of the scalar field σ, the ρ-meson mass results from spontaneous breaking of chiral symmetry. This collective state is described by the antisymmetrical tensor field $\rho_{\mu\nu}$. It is interesting that J.Gasser and H.Leutwyler used the very Lagrangian to describe the ρ-meson while estimating the effect of resonance states on the low-energy structure of Green functions [9] (see also [10]). The wave equation associated with Lagrangian (7) has the form

$$\left\{ \begin{array}{l} \dot{\Pi}_i + \partial_i\partial_j\rho_{j0} - m_\rho^2\rho_{0i} = 0 \\ \partial_j\Pi_i - \partial_i\Pi_j - m_\rho^2\rho_{ji} = 0, \end{array} \right. \tag{9}$$

where the canonical momentum Π_i is $\Pi_i = -\partial_\mu\rho_{\mu i}(i,\, j = 1,\, 2,\, 3)$. According to these equations, only the ρ_{0i} field components oscillate. The components ρ_{ij} are frozen. At $m_\rho = 0$ (or $m = 0$) equation (9) possesses symmetry under the following transformations: $\rho_{i0} \to \rho_{i0} + \partial_i\theta(\vec{x}), \vec{\nabla}^2\theta(\vec{x}) = 0$. One can use this freedom imposing the gauge fixing requirement $\partial_i\rho_{i0} = 0$. In this case the field keeps only transverse degrees of freedom, and the vector mode is fully frozen. It means that if $F(q^2)$ is the vector meson coupling constant for virtual momentum q_μ then it reduces to zero in the limit $q^2 \to 0$ (or $m \to 0$). Decoupling of the vector field occurs. D.Caldi and H.Pagels already supposed the existence of this dormant state earlier [11].

To proceed, we must fix the parameters of the model. For direct comparison with the empirical numbers we have used the values for the pion decay constant, f_π, and the pion mass, m_π, close to their physical values, $f_\pi \simeq 93$ MeV and $m_\pi \simeq 139$ MeV, respectively. Within the purely fermionic NJL model [12], a similar set of parameters has already been determined. For the case at hand, we used $G_1 = 7.74\,\mathrm{GeV}^{-2}, \Lambda = 1.0$ GeV and $\widehat{m} = 5.5$ MeV. With these parameters, we find $f_\pi = 93.1\,\mathrm{MeV}, m_\pi = 139\,\mathrm{MeV}, m = 241.8\,\mathrm{MeV}$ and $m_\rho = 765$ MeV. Let us note that we should not introduce the new parameters to describe the mass of the vector particles.

Let us discuss the main features of the (ρ,ω)–(π,γ) interactions in the model under consideration. As far as calculations are concerned, we shall not go into detail leaving it to a future lengthier paper. The relevant effective Lagrangian contains the following terms

$$\begin{aligned} L_I &= \frac{e}{f_\rho}\partial_\mu A_\nu(\rho_{\mu\nu}+\frac{1}{3}\omega_{\mu\nu}) + f_{\rho\pi\pi}\varepsilon_{ijk}\rho^i_\mu\pi^j\partial_\mu\pi^k - \frac{ef_{\rho\pi\gamma}}{2f_\rho f_\pi}\varepsilon^{\mu\nu\lambda\sigma}\partial_\mu A_\nu\partial_\lambda\rho^i_\sigma\pi^i \\ & -\frac{f_{\omega 3\pi}}{f_\rho f_\pi^3}\varepsilon^{\mu\nu\lambda\sigma}\varepsilon_{ijk}\omega_\mu\partial_\nu\pi^i\partial_\lambda\pi^j\partial_\sigma\pi^k + \ldots, \end{aligned} \tag{10}$$

where we have introduced a symbol A_μ for an electromagnetic field[1] and left out pieces of no concern to us here.

The constants of the $\rho\to\pi\pi$ and $\rho\to\gamma$ transitions have the form

$$f_{\rho\pi\pi} = \frac{\sqrt{3G}m^2(m_\rho^2-2m_\pi^2)}{4\pi^2 f_\pi^3}, \qquad \frac{1}{f_\rho} = \frac{\sqrt{G}f_\pi}{\sqrt{3}}. \tag{11}$$

Hence we derive the relation

$$m_\rho^2 - 2m_\pi^2 = \frac{4\pi^2 f_\pi^4}{3m^2} f_{\rho\pi\pi}f_\rho \tag{12}$$

which generalizes the remarkable KSRF result $m_\rho^2 = 2f_{\rho\pi\pi}f_\rho f_\pi^2$ to the case of broken chiral symmetry $\widehat{m}\neq 0$. Ignoring the pion mass, one understands the origin of the mysterious two

$$2 \sim \frac{(2\pi f_\pi)^2}{3m^2} = 1.96. \tag{13}$$

Relation (11) can be used to deduce

$$\frac{f_{\rho\pi\pi}f_\rho}{4\pi} = \frac{3m^2(5m^2-m_\pi^2)}{8\pi^3 f_\pi^4} = 2.55 \tag{14}$$

The available data [13] require $f^2_{\rho\pi\pi}/4\pi = 2.9$ from $\Gamma(\rho\to\pi\pi) = 149$ MeV and $f_\rho^2/4\pi = 2.0$ from $\Gamma(\rho\to e^+e^-) = 6.8$ keV. Combining these data we obtain $(f_{\rho\pi\pi}f_\rho/4\pi)^{exp} = 2.41$. Agreement in (14) is even better because the ρe^+e^- coupling $f_\rho^2/4\pi$ is corrected from 2.0 to 2.4.

[1] It is traditional to include the interaction with the electromagnetic field via the standard replacement of the derivative ∂_μ in Lagrangian (3) by the covariant $\partial_\mu - ieQA_\mu$.

The dimensional parameter $\sqrt{G}$ can play a special role. Taking

$$\sqrt{G} = \frac{\sqrt{2}\pi f_\pi}{m\sqrt{5m^2 - m_\pi^2}} \simeq \sqrt{\frac{2}{5}}\frac{\pi f_\pi}{m^2} = 0.62\,\mathrm{fm}, \tag{15}$$

we get the universality condition $f_{\rho\pi\pi} = f_\rho$. The slight discrepancy observed in $f_{\rho\pi\pi}$ and f_ρ can be attributed to deviation from the value (15). It should be pointed out that the value of this parameter has no influence on the form of the KSRF relation. This is different from the hidden local symmetry approach [14], where the requirement of universality leads to the KSRF relation and *vice versa*.

We also note that the coupling constant $f_{\rho\pi\gamma}$ is equal to

$$f_{\rho\pi\gamma} = \frac{m_\rho^2 + m_\pi^2}{8\pi^2 f_\pi^2}. \tag{16}$$

Therefore the leading coupling that allows these transitions is $\mathcal{O}(p^4)$. This fact is known from the paper [10].

We have a very interesting situation with the $\omega \to \pi\pi\pi$ decay. An early investigation, published in 1962 by M.Gell-Mann, D.Sharp and W.Wagner (GSW) [15], took the point of view that the ρ-pole term completely dominates in this process. In our model there is no coupling that would induce this transition at the quark one-loop level. In this leading approximation the $\omega\rho\pi$ vertex is equal to zero and can appear only at the level of mesonic loops at higher order of momentum expansion. Instead of the GSW model, we can describe this decay allowing for the only contact term in $\omega \to 3\pi$. The constant value of $f_{\omega 3\pi}$ gives good agreement with the Dalitz plot of these decays. Our calculations of the box diagrams show that

$$f_{\omega 3\pi} = \frac{3m^2}{4\pi^2 f_\pi^2}\left(1 + \frac{m_\omega^2 + 3m_\pi^2}{12m^2}\right). \tag{17}$$

Therefore, the resulting width $\Gamma(\omega \to \pi^+\pi^0\pi^-) = 6.0$ MeV is in satisfactory agreement with the experimental value $7.49 \pm 0.09 \pm 0.05$ MeV [13].

A final remark: The proposed model indicates that vector mesons, like pseudoscalar ones, are collective excitations of the quark sea. This statement is grounded on two significant consequences of the approach in question. One is a surprisingly good description of the ρ-meson mass, the other is the KSRF relation which arises naturally here and whose character has remained obscure so far.

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